

## Mathematical Methods of Physics II

|                                       |                      |                                           |               |
|---------------------------------------|----------------------|-------------------------------------------|---------------|
| Module Code: Phys-M2031;              | EtCTS of Course: 5 ; | Course Status:                            | Compulsory    |
| <b>Course Title :</b>                 |                      | <b>Mathematical Methods of Physics II</b> |               |
| Course Code:                          | Phy2032;             | Credits Hours:                            | 3             |
| Mode of delivery:                     | Full Semester;       | Weeks required:                           | 16            |
| Prerequisite(s):                      |                      | Co-requisite(s):                          |               |
| Academic Year:                        | 20____/____ ;        | Year/Semester:                            | II/ II        |
| Students' College/Faculty:            | _____;               | Department:                               | Physics       |
| Program:                              | Undergraduate        | Enrollment:                               | _____         |
| Instructor's Name (Coordinator) _____ |                      |                                           |               |
| Address:                              | Email: _____         | Block No. _____;                          | Rm. No. _____ |
| Class Hours: _____                    |                      |                                           |               |

### Course Rationale

This course aims to give learners a deeper understanding of and greater competence in some central mathematical ideas and techniques used in Physics with the emphasis on practical skills rather than formal proof. Students will acquire skills in some key techniques related directly to the advanced courses they will meet in their final year.

### Learning Outcomes

Upon completion of this course students should be able to:

- solve partial differential equations by separation of variables;
- calculate eigenvalues and eigenvectors and apply the techniques to physical problems;
- use basis vectors to transform differential operator equations to matrix form and hence apply eigen equation techniques;
- obtain approximate solutions to differential equations through the use of perturbation theory.
- develop analytical and numerical skills in mathematics;
- formulate problems logically;
- present and justify mathematical techniques and methods;

### Course Description

**Vectors and Matrices** algebra of vectors, basis vectors and components, vector spaces, matrix algebra, numerical methods for matrices, coordinate transformation, Four-vectors, eigen value problem

**Vector Calculus** time derivatives of vectors, fluid kinematics, fluid dynamics, fields and the gradient, fluid flow and the divergence, circulation and the curl, conservative forces and the Laplacian, electric and magnetic fields, vector calculus expressions and identities. **Complex Variables:** functions of a complex variable, differentiation and integration, cauchy integral formula and Laurent Expansion; Singularities, poles and residues, applications

**Partial Differential Equations:** introduction to PDEs, the wave equation, Laplace's equation, Orthogonal functions and the Sturm-Liouville problem; **Special Functions:** Legendre, Bessel and Hermite Equations

### Course Outcomes

Upon completion of this course students should be able to:

- manipulate vectors and matrices and solve systems of simultaneous linear equations;

- perform basic arithmetic and algebra with complex numbers;
- use the ideas of singularities and poles to evaluate line integrals.
- apply differential operators to vector functions;
- apply Stokes's and Gauss's theorems;
- use basis vectors to transform differential operator equations to matrix form and hence apply eigen equation techniques;
- obtain approximate solutions to differential equations through the use of perturbation theory.
- use the method of separation of variables to solve PDEs;
- solve PDEs in various coordinate systems;
- use numerical techniques for solving Laplace's equation
- Analytical and numerical skills in mathematics;
- Logical formulation of problems;
- Presentation and justification of techniques and methods;
- Group work - students are encouraged to work co-operatively together and with the demonstrators to solve guided problems.

### Course Outline: ECTS (5)

| No | Topic                                 | Lecture (hrs) | Tutor (hrs) | Home (hrs) | T.Load (hrs) |
|----|---------------------------------------|---------------|-------------|------------|--------------|
| 1) | Vectors and Matrices                  | 10            | 3           | 15         | 28           |
| 2) | Vector Calculus                       | 12            | 4           | 19         | 35           |
| 3) | Complex Variables                     | 8             | 3           | 16         | 27           |
| 4) | Partial Differential Equations (PDEs) | 16            | 5           | 24         | 45           |
|    | Total                                 | 46            | 15          | 75         | 135          |

### Method of Teaching

Presentation of the course is through lecture, a related guided problems section with demonstrator assistance and additional assessed coursework. Online learning resources.

## Tentative Time Breakdown of Lecture Topics

| Date            | Topics                                                                                                                                                        | Pedagogical Approaches                                                                     | Teachers' Tasks/Activities                                                                                                                                     | Students' Tasks/Activities                                                                                        |
|-----------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------|
| Week 1          | <b>Vectors and Matrices</b><br>• Algebra of Vectors<br>• Basis Vectors and Components<br>• Vector Spaces<br><b>Worksheet and Assignment I</b>                 | Lecture,<br>Problem solving in group and at individual level,<br>Online learning resources | Give worksheet and Assignment, Presentation of lecture, Demonstrate problem solving, Give exercises and monitor the process, Solve worksheets during tutorials | Take notes, Ask questions, Solve exercises, Work in group and individual, Work their assignment, Prepare for test |
| Week 2          | • Matrix Algebra<br>• Numerical Methods for Matrices<br>• Coordinate Transformations                                                                          |                                                                                            |                                                                                                                                                                |                                                                                                                   |
| Week 3          | • Four- Vectors<br>• The Eigenvalue Problem                                                                                                                   |                                                                                            |                                                                                                                                                                |                                                                                                                   |
| <b>Test I</b>   |                                                                                                                                                               |                                                                                            |                                                                                                                                                                |                                                                                                                   |
| Week 4          | <b>Vector Calculus</b><br>• Time derivatives of vectors<br>• Fluid kinematics and dynamics<br>• Fields and the Gradient<br><b>Worksheet and Assignment II</b> | Lecture,<br>Problem solving,<br>Online learning resources                                  | Give worksheet and Assignment, Presentation of lecture, Demonstrate problem solving, Give exercises and monitor the process, Solve worksheets during tutorials | Take notes, Ask questions, Solve exercises, Work in group and individual, Work their assignment, Prepare for test |
| Week 5          | • Fluid flow and the Divergence<br>• Circulation and the Curl<br>• Conservative Forces and the Laplacian                                                      |                                                                                            |                                                                                                                                                                |                                                                                                                   |
| Week 6          | • Electric and Magnetic Fields<br>• Vector Calculus Expressions and Identities                                                                                |                                                                                            |                                                                                                                                                                |                                                                                                                   |
| <b>Test II</b>  |                                                                                                                                                               |                                                                                            |                                                                                                                                                                |                                                                                                                   |
| Week 7          | <b>Complex Variables</b><br>• Functions of a Complex Variable<br>• Differentiation and Integration<br><b>Worksheet and Assignment II</b>                      | Lecture,<br>Problem solving,<br>Online learning resources                                  | Give worksheet and Assignment, Presentation of lecture, Demonstrate problem solving, Give exercises and monitor the process, Solve worksheets during tutorials | Take notes, Ask questions, Solve exercises, Work in group and individual, Work their assignment, Prepare for test |
| Week 8          | • Cauchy Integral Formula and Laurent Expansion<br>• Singularities, Poles and Residues                                                                        |                                                                                            |                                                                                                                                                                |                                                                                                                   |
| Week 9          | • Applications                                                                                                                                                |                                                                                            |                                                                                                                                                                |                                                                                                                   |
| <b>Test III</b> |                                                                                                                                                               |                                                                                            |                                                                                                                                                                |                                                                                                                   |

| Date    | Topics                                                                                                        | Pedagogical Approaches                              | Teachers' Tasks/Activities                                                                                                                                                                            | Students' Tasks/Activities                                                                      |
|---------|---------------------------------------------------------------------------------------------------------------|-----------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------|
| Week 10 | <b>Partial Differential Equations (PDEs)</b><br>• Introduction to PDEs<br><b>Worksheet and Assignment III</b> | Lecture, Problem solving, Online learning resources | Give worksheet and Assignment, Presentation of lecture, Demonstrate problem solving, Give exercises and monitor the process, Solve worksheets during tutorials, Prepare topics for group presentation | Take notes, Ask questions, Solve exercises, Work in group and individual, Work their assignment |
| Week 11 | • Wave Equation Revisited                                                                                     | Presentation                                        |                                                                                                                                                                                                       | Prepare for test, Work on their assigned presentation                                           |
| Week 12 | • Laplace's equation                                                                                          |                                                     |                                                                                                                                                                                                       |                                                                                                 |
| Week 13 | • Orthogonal functions and the Sturm-Liouville Problem;                                                       |                                                     |                                                                                                                                                                                                       |                                                                                                 |
| Week 14 | • Special Functions                                                                                           |                                                     |                                                                                                                                                                                                       |                                                                                                 |
| Week 15 | <b>Revision and self study</b>                                                                                |                                                     |                                                                                                                                                                                                       |                                                                                                 |
| Week 16 | <b>Final Exam</b>                                                                                             |                                                     |                                                                                                                                                                                                       |                                                                                                 |

### Assessment

| No | Type of Assessment           | Time       | Weight |
|----|------------------------------|------------|--------|
| 1  | Test I                       | Week 3     | 10%    |
|    | Test II                      | Week 6     | 10%    |
|    | Test III                     | Week 9     | 10%    |
| 2  | Assignment I                 | Week 1     | 5%     |
|    | Assignment II                | Week 4     | 5%     |
|    | Assignment III               | Week 7     | 5%     |
| 3  | Class activity and home work | All weeks  | 5%     |
| 4  | Final exam                   | Final week | 50%    |
|    |                              | Total      | 100%   |

### COURSE PROGRESS EVALUATION FOCUS

Course evaluation will be through feedback received from students in the lecture, tutorial programmes and test results and through an anonymous student survey which should be conducted every month.

### KEY STUDENT-RELATED POLICIES

Students should read the relevant sections in the textbook and/or reference materials and do the assignments on time. Practice with solved problems and come to office hours to get concepts clarified. Review and extra problems will be given through worksheets. Students are also expected to have worked through the problems in the worksheets before the tutorial sessions. Attendance at lectures and tutorials is expected for all students. Attendance records will be taken at all times. It is the students chance to ask questions, solve problems and work in team.

### Recommended References

#### Course Textbook

Spiegel M.R., *Advanced Mathematics for Engineers and Scientists*, Schaum Outline Series, McGraw-Hill, (1971).

### References

1. Arfken G.B. and Weber H.J., *Mathematical methods for physicists* (6<sup>th</sup> ed.), Academic Press, 2006.
2. Spiegel M.R., *Advanced Mathematics for Engineers and Scientists*, Schaum Outline Series, McGraw-Hill, 1971.

3. Stroud K.A., *Engineering Mathematics* (5<sup>th</sup> ed.), Paulgrave, 2001.
4. Donald A. McQuarric, *Mathematical Methods for Scientists and Engineers*, University Science Books, 2003.
5. Lambourne R. and Tinker M. *Further Mathematics for the Physical Sciences*, Wiley, 2000.
6. Mathews J. and Walker R.L., *Mathematical Methods of Physics*, 2<sup>nd</sup> ed., 1970.